Philip Kahn – 02/20/2009
Tri-Valley Stargazers & Astronomical Society

PHYSICS AROUND BLACK HOLES

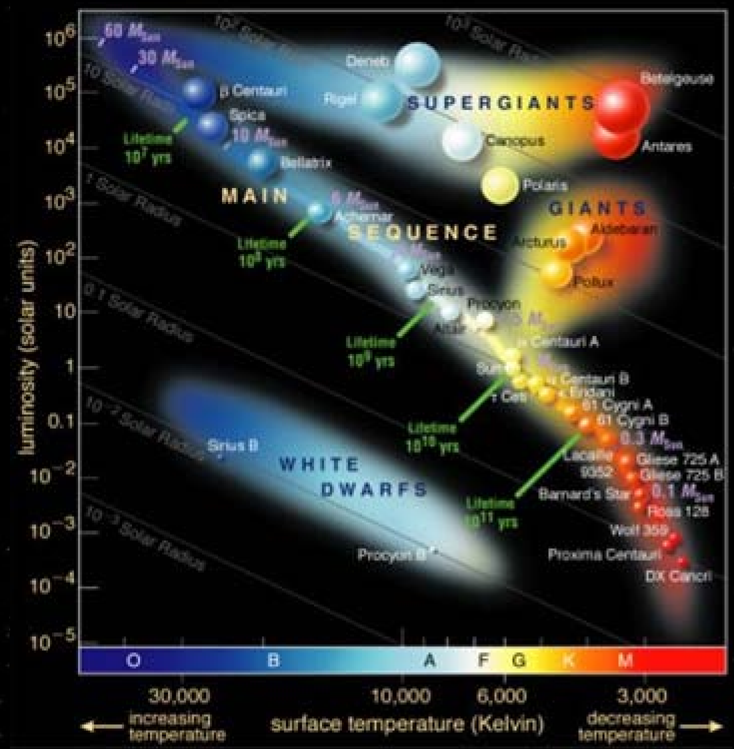
Know Thy Enemy

- Black holes are:
  - Infinitesimally small
  - Asymptotically dense
    - $\rho \propto M/L_P^3$
- Black holes do:
  - Heavily distort spacetime near them
  - Act gravitationally normal outside the progenitor star's radius

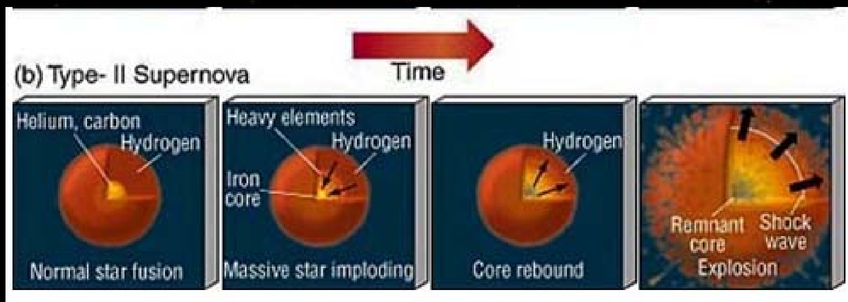
- Black holes are ***not***:
  - "Cosmic vacuum cleaners"
  - Necessarily black
- Black holes do ***not***:
  - Inherently do anything special past a few Schwarzchild radii
  - Necessarily tear objects apart near the event horizon

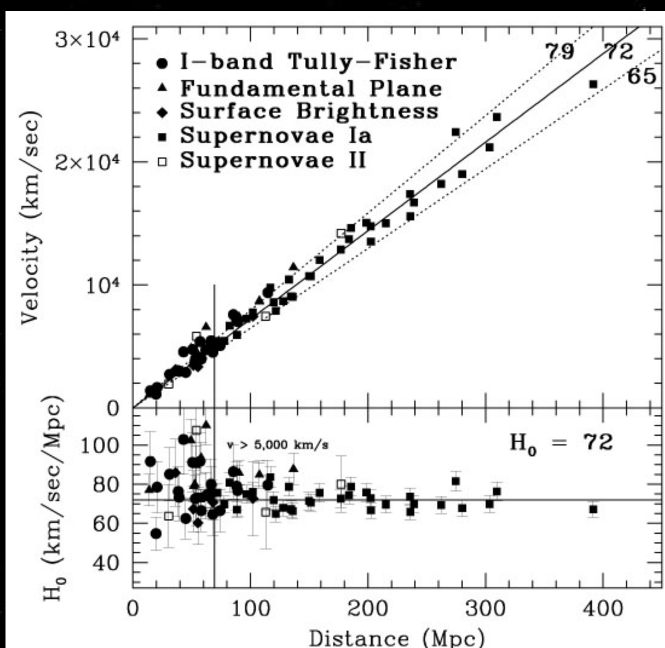
Stellar Death

- Stellar death is a powerful, yet rare event. Novae and supernovae can shine with $10^9 - 10^{11}$ solar luminosities, outshining galaxies
- Rarely, with very large stars, stellar mass ejection leaves sufficient mass behind that the stellar core is still multiple solar masses.



Stellar Death





- This remainder is violently compressed by gravity and a bouncing hydrogen fusion shell.
- The resulting compression and gravity results in the formation of a compact object.

Black Hole Birth

- Black holes are the most extreme form of compact object. They occur when all other support forms fail.
  - White dwarfs, the least extreme form, are no longer supported by electromagnetism but instead supported by a quantum pressure – "electron degeneracy pressure" – that is the same principle leading to discrete electron orbitals in atoms.
  - The next stage is a neutron star, where EDP fails, and via ($p^+ + e^- \longrightarrow n + \gamma + \nu_e$) the stars mass collapses down to a mass of neutrons all in a single quantum state, as a superfluid.

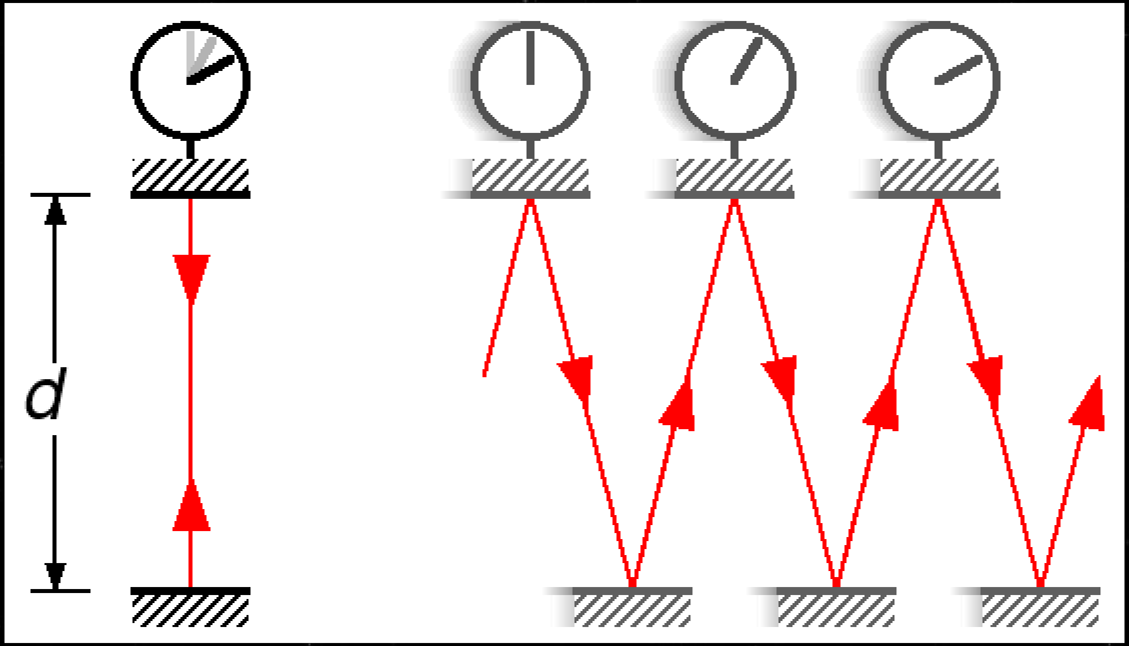
Spacetime and Relativity

- Space and time were two distinct concepts until Einstein's 1905 paper on special relativity.
- The spacetime interval is a way of measuring invariant distance in relativity, and is directly related to proper time.
  - Algebraically:
    - $(\Delta\tau)^2 = -(\Delta s)^2 = -(\Delta t)^2 + (\Delta x)^2 + (\Delta y)^2 + (\Delta z)^2$
  - Tensor index notation:
    - $(\Delta\tau)^2 = -(\Delta s)^2 = \eta_{\mu\nu}\Delta x^\mu \Delta x^\nu$ where $\eta_{\mu\nu} = \begin{pmatrix} -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$

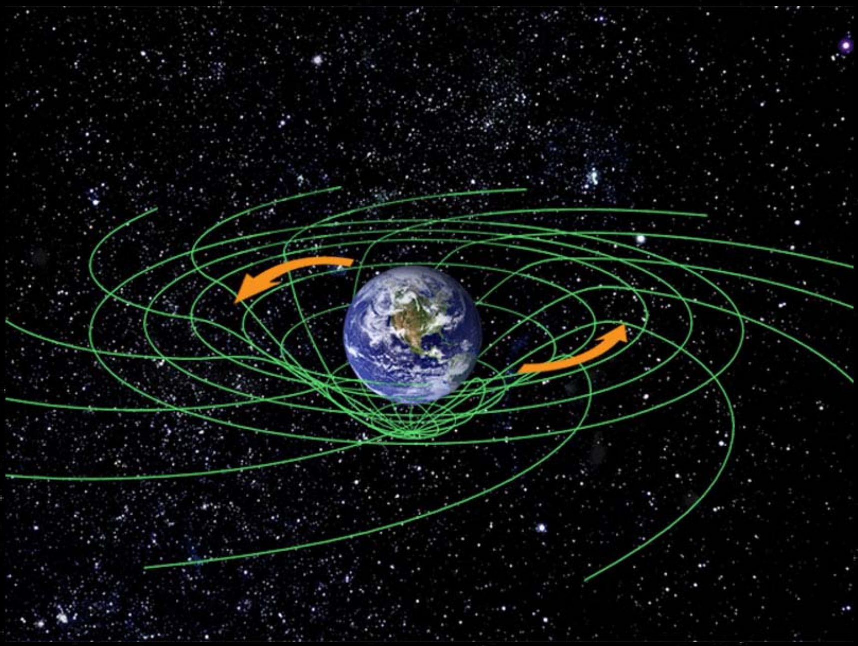
Spacetime and Relativity

The result of this is that with special relativity, it becomes apparent that **relative** motion between two individuals is important, and further, information travel is limited to moving at light speed . The factor that describes this is the "gamma factor" (in natural units):

$$\gamma = 1/\sqrt{(1-v^2)}$$



Effects of Gravity on Spacetime



- In 1915, Einstein finished the generalization of his 1905 theory.
  - The formalism gave unique predictions, based on distorting the geometry of spacetime.

The Einstein Equations

The 1915 theory expressed space and time as a unified whole, providing metric solutions for various mass-energy distributions.

$$
\begin{aligned}
G_{\mu\nu} &\equiv R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R \\
&= 8\pi G T_{\mu\nu} \\
R_{\mu\nu} &= \Gamma^{\alpha}_{\ \mu\nu,\alpha} - \Gamma^{\alpha}_{\ \mu\alpha,\nu} + \Gamma^{\alpha}_{\ \beta\alpha}\Gamma^{\beta}_{\ \mu\nu} - \Gamma^{\alpha}_{\ \beta\nu}\Gamma^{\beta}_{\ \mu\alpha} \\
R &\equiv g^{\mu\nu} R_{\mu\nu}
\end{aligned}
$$

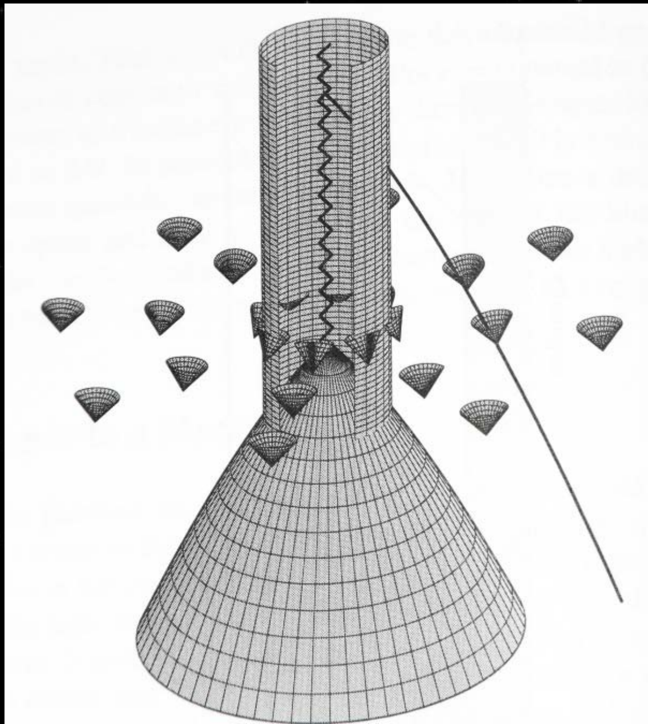
Schwarzschild Geometry

$$
ds^2 = -\left(1 - \frac{2M}{r}\right) dt^2 + \left(1 - \frac{2M}{r}\right)^{-1} dr^2 + r(d\theta^2 + \sin^2\theta d\phi^2)
$$

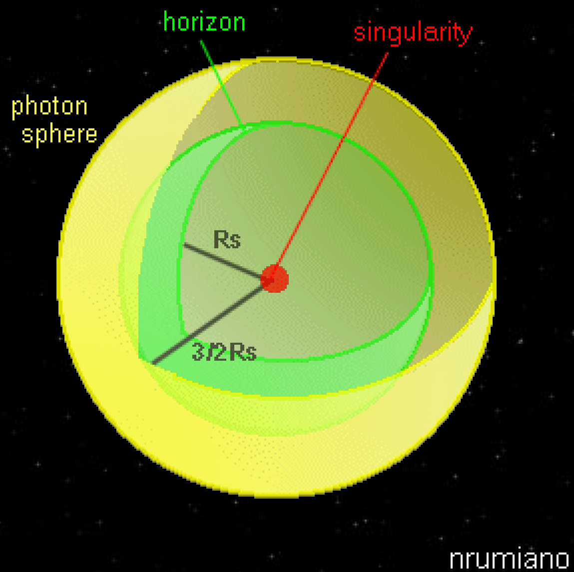
The first solution to the EFEs, the Schwarzschild metric, had an interesting mathematical singularity at $r=2GM/c^2$

The Schwarzschild Radius

- The Schwarzschild radius, more commonly known as the "event horizon", is the distance beyond which velocities higher than *c* would be needed to allow information to escape.
  - Though this is a mathematical singularity in the Schwarzschild coordinates, and it hints at some interesting behavior, it is continuous and other coordinate systems don't have the mathematical singularity.
    - Eddington-Finkelstein
    - Kruskal-Szekeres
    - Kerr



Physics at Small R_s



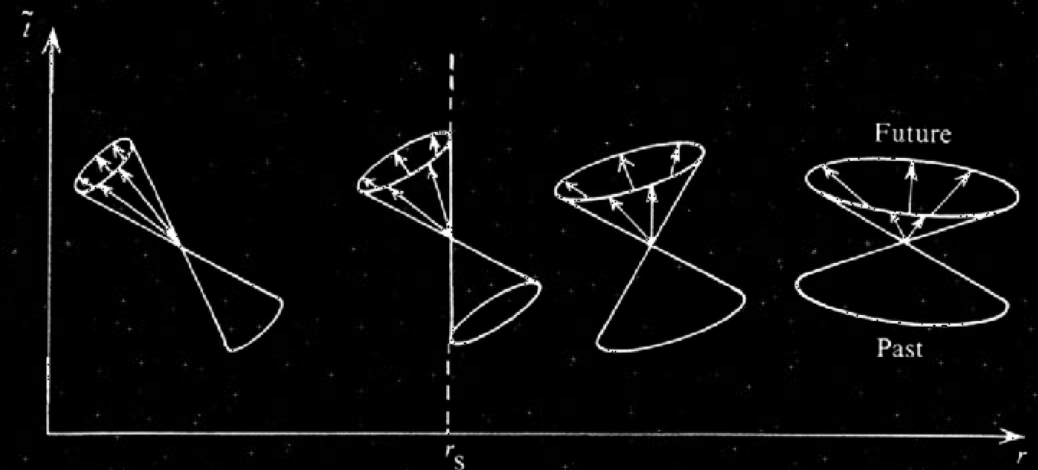
- Several interesting things happen near the event horizon.
  - Photon sphere
  - Color shifting
  - Unstable orbits
  - Time dilation

Inside the Event Horizon

- The event horizon actually enables the black hole to be the highest entropy structure in the universe.
  - While it may seem that a black hole is “ordered”, in fact the presence of the event horizon and the “no-hair” principle means that the highest number of microstates (or configurations) are available to a specific mass within the event horizon. Since entropy is defined by the number of microstates, a black hole is the most entropic structure in the universe.

Inside the Event Horizon

Inside the event horizon, the strong warping of spacetime tilts an observer's light cone by 90° (π/2 rad). This means that there is an equal probability for your motion in time to occur "forward" or "backward". Physics covers its bets, however – residing inside the event horizon, you cannot violate causality, as no information can leave.



Inside the Event Horizon

- Of course, in side the event horizon lives the "singularity"
  - Singularities cannot occur outside of event horizons – there is no such thing as a "naked singularity".
- The singularity is so named because physics breaks down around it. It becomes too small for GR to handle, predicting infinities; and QM cannot handle it, as it is too massive so spacetime curvature is important.

Quantum Effect of Black Holes

- Black holes are subject to quantum effects, too.
  - Hawking radiation
  - Maximal entropy is proportional to area

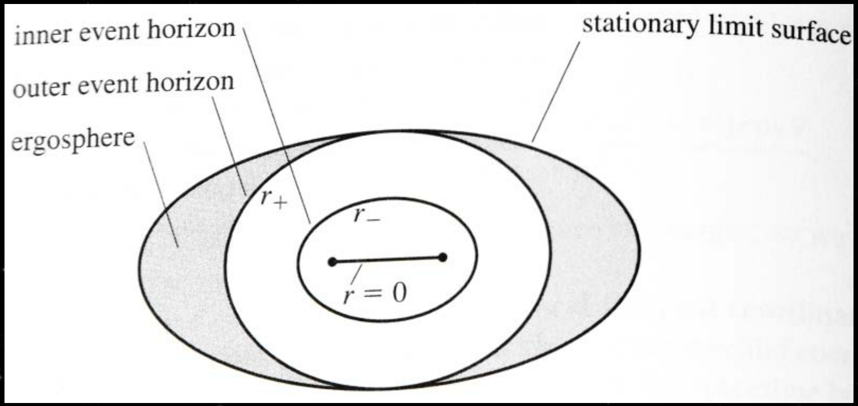
Hawking Temperature

$$k_B T = \frac{c^3 \hbar}{8\pi G M}$$

Black Hole Entropy

$$S_H = \frac{k_B}{4\hbar} A$$

The Simplest Case

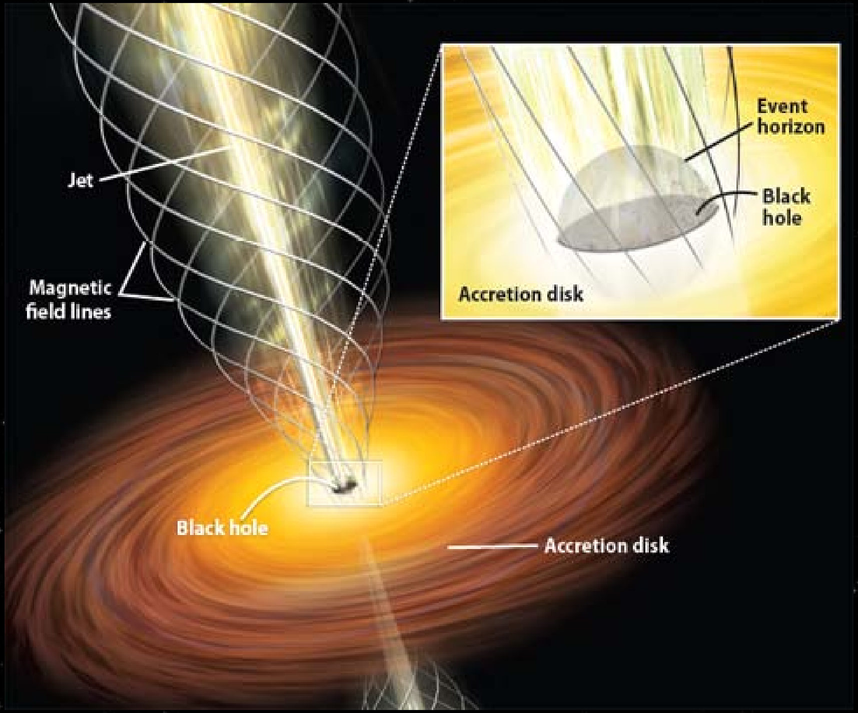


- Everything discussed is for stationary black holes. If they rotate, the physics becomes far more complicated.
  - The Ergosphere – last stationary observer
  - Gravitational waves
  - Asymmetric horizon

Effects of Black Holes on Large-Scale Structure

- Black holes don’t have a unique effect on large scale structure, because the interesting physics happens only at low R_s.
- However, since black holes can encompass a great deal of mass in a very small area, they can form galactic nuclei, and may be the seeds for galaxies.
- Additionally, since black holes will accrete and “feed” on material, they can clear the area close to them of objects in unstable orbits, “selecting” only neat galactic forms with stable orbits.

Observational Effects of Black Holes



- Black holes are often quite luminous.
  - Infalling matter rubs against itself in a characteristic manner, heating it up and making it emit photons at various wavelengths, including visible.
  - Infalling matter can also be accelerated and ejected out at relativistic speeds.

Sources

- *Modern Cosmology*. Dodelson, S. 2003. ISBN 0-12-219141-2
- *Spacetime and Geometry*. Carroll, S. 2004. ISBN 0-8053-8732-3
- *Gravity*. Hartle, J. 2003. ISBN 0-8053-8662-9